USANDO ABP EM TURMAS DE CÁLCULO

Using the PBL method in Calculus classes

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Abstract

Different methodologies have been used in Calculus teaching. In particular, active methodologies such as problem-based learning have been adopted. However, compared to other courses, this methodology has been little used in mathematics courses in higher education. We investigate here a little of this trajectory in a particular case and present the result of a research done in two classes of Calculus, under the methodology of comparative analysis: one with traditional teaching and another using an active methodology. Results point to a need for pedagogical and methodological innovation in Calculus courses.

Keywords Calculus. Teaching experiment. PBL. Feedback.

Resumo

Metodologias diferentes tem sido usadas no ensino de Cálculo. Particularmente, metodologias ativas tais como aprendizagem baseada em problemas tem sido adotada. Contudo, essa metodologia tem sido pouco usada em cursos de matemática. Investigamos aqui um pouco dessa trajetória e apresentamos o resultado de uma pesquisa feita em duas turmas de Cálculo 3, sob a metodologia de análise comparativa: uma em que foi aplicada a metodologia ABP e outra usando o tradicional ensino. Os resultados apontam para uma necessidade de inovação pedagógica e metodológica em cursos de Cálculo.

Palavras-chave Cálculo. Experiência de Ensino. ABP. Feedback.

1 INTRODUCTION

The history of teaching calculus contains many efforts to improve students' conceptual understanding, and the ensuing conflicts and controversies over the success of methods that claim to improve students' conceptual learning while continuing to develop students' procedural proficiencies (Schoenfeld, 2004).

Learning is more than the acquisition of the capacity to think. It is the acquisition of many specialized abilities to think about several things (Vygotsky, 1978). This means that the goal is not merely to learn by learning, to memorize, or to reproduce certain knowledge, but it is hoped that the acquired learning will serve beyond the educational environments. Vygotsky's theory suggests that by interacting at a high level, the learner will internalize the knowledge and values that he or she uses, always through interaction processes, whether or not you can identify them the moment you use. The internalization and the development of skills enable the learner to be able to start a new cycle of learning at a higher cognitive level.

The problem-based learning methodology consists of an instructional method that makes use of real-life problems, stimulating the development of critical thinking. This teaching method also helps with problem solving skills and learning concepts that integrate program content. We follow the precept of Howard Barrows (1986), one of the pioneers in the development of PBL.

The PBL methodologies differs from the traditional teaching. The assumption is that in a traditional lecture one person is standing up at the front addressing a large audience. They will speak for close to an hour and a half, from notes that they have written themselves, on a single topic, possibly allowing questions near the end. They are likely to use some kind of visual aids, and students will be silent, taking notes.

For many researchers, ideal learning is practiced in small groups. Members work together to learn how to solve the problem and acquire collaborative learning skills. The groups investigate the problems, coordinate their efforts to achieve the collective goal, and collaborate in writing and presenting the findings. Students participate in a learning cycle, discuss and receive feedback from group and teacher colleagues. Even in large classes, it is possible to apply a teaching methodology that encourages students and improves performance (Code et al., 2014).

The results of empirical studies using the PBL method in higher education show that students perform better in applying their knowledge. They also have positive effects on students' skills development (Yusof e*t al*., 2005; Dochy *et al.*, 2003).

What could be of help to improve teaching, motivate students, and reduce high dropout rates? An active methodology of teaching and learning may be the answer. This methodology can be used from primary level to higher education.

In several universities about 50% of students take the Calculus courses at least twice. This rate can be improved with the use of PBL. Better yet, using this active methodology, students will retain deeper knowledge.

The history of the use of PBL methodology in undergraduate courses in Brazil is recent. Increasing adoption of this method of teaching has been made in the last fifteen years. In mathematics courses, this growth has occurred in the last five years.

Experiments with the PBL method point to the need for change of habit, both students and teachers. This is due to requirements related to the method for dealing with active learning strategies and demand a greater willingness to autonomous learning.

Some methodology used in conjunction with traditional lectures is a good start. An active learning pedagogy can be used even in a large class with several students. For example, students can be assign to read and experiment in preparation for class (Code et al., 2014). Interventions in early undergraduate mathematics would be better for student progress in their academic life.

University and scientific training must offer society professionals who are able to effectively develop their daily work activities by adding conceptual, technical, human, political and critical domains to them. The approach guided by the PBL includes these features. It adopts constructivist assumptions that attribute learning to the action of the subject on the objects of knowledge, their practical experiences and the learning contexts, establishing a dialogical relation between subject and object.

This article has a clear goal. Briefly review the use of active methodologies in teaching mathematics, particularly in higher education in Brazil. Comparing traditional teaching with the active methodologies raises important points. Then we present a result in two classes. There are few innovative methods for teaching mathematics in higher education. Some topics of Calculus, particularly the third, have not been addressed using active methodologies. This was one of the reasons that the experiment presented at the end was done with the course of Calculus 3. It is interesting to see the students feedback. The obtained data were analyzed quantitatively to give summative statistics, and qualitatively to obtain contextualized information with more nuances.

2 PROBLEM SOLVING IN BRAZIL: A BRIEF HISTORY

The first experiences of adoption of PBL methodology in Brazil occurred in the 1990s. This methodology was used in medical education curricula in two Brazilian states: Paraná (University of Londrina, UEL, in 1995 and 1998) and São Paulo (University of Marília, UNIMAR, in 1997). The medical course of the School of Medicine of Marília FAMEMA adopted the PBL, implemented the new curriculum in a gradual way, starting in 1997, and in 2002 graduated the first group of students formed by the ABP methodology (Decker & Bouhuijs, 2009).

Despite the increasing use of active methodologies in Brazilian higher education, there are still few courses and professors that use it. Particularly, in mathematics courses the use of PBL in teaching is still quite small.

In Brazil, the calculus courses are divided into three semesters:

- Calculus 1: Limits, Derivatives and Integration.
- Calculus 2: Polar Coordinates, Vector Functions, Sequences and Series.
- Calculus 3: Multiple Integration and Vector Calculus (Line and Surface Integrals, Green's Theorem, the

Divergence and Stokes's Theorem).

Each course lasts for one semester. There are few changes between a university and another. For example, some often provide the pre-calculation course. Some Brazilian higher education professors have struggled to apply active methodologies in mathematics teaching.

Baracat, Witkowski and Cutri (2013), professors at the School of Engineering Maua, presented a project for using PBL in topics of Calculus 2 and 3. This happened at the Eleventh LACCEI - Latin American and Caribbean Conference for Engineering and Technology, in Cancun Mexico, 2013. They did something interesting. Related mathematical topics that students are learning, they conceived some typical engineering problems for use with the PBL methodology. For example: the study of the possibility of installing a small hydroelectric plant and then determine the hydraulic power fall, and therefore the type of turbine to be installed.

Débora Souza, professor at the UNASP University Center, in the Brazilian state of São Paulo, presented a project for using PBL in some topics of Calculus 1 (presented at the PBL2016 International Conference). She has formulated an interesting PBL problem related to citrus industry, more particularly the orange fruit. The problem has good applicability because Brazil is the country that more exports this fruit and also accounts for about 60% of world production of orange juice.

Examples of the use of active methodologies in mathematics teaching in higher education are not many. In several universities about 50% of students take the courses of Calculus 1 at least twice. Many professors and managers of universities recognize the need for some change to improve these rates. Some have started to make changes. But in the case of mathematics teaching and learning, the changes are still small. Many professors prefer to give explanatory classes throughout a course. Or, teach with power point slides. And then, they apply about two to four individual tests and exams to check the knowledge of the student. That is, they remain in the comfort zone. The biggest problem is not to give lessons with slides or explanatory classes, but do nothing to improve teaching and learning.

The experience of the University of Delaware has been of great benefit to the Brazilian Universities (Groh et al, 1997). In the early 2000s, Brazilian professors from several area began to be trained by University of Delaware professors to use the PBL methodology. This training has taken place in several universities in Brazil. In 2015 and 2016 UNIFEI had the opportunity to receive professors from Delaware for this purpose.

Despite partnerships with foreign universities, many professors have not been willing to change teaching methodologies. Most still prefer a normal or 'traditional' lecture. We hope that more professors adopt new methodologies in teaching for the benefit of the students.

3 METHODOLOGY

According to Collier (1993) “Comparison is a fundamental tool of analysis. It sharpens our power of description and play a central role in concept-formation by bringing into focus suggestive similarities and contrast among cases”.

The comparative study allows us to better understand a phenomenon under study. In the present work a report of a comparative study between two classes was done, one using the PBL and another without the use of this method. Bruhn (2009) does a study comparing two examples in which the changed perception of the meaning and uses of comparative education and concluded that it was achieved a successful cross-national research emphasizing its potential for extending fundamental educational knowledge on a global basis. In other words, Maths education depends on how teachers work it. Jablonka et al. (2016) show that in Europe, comparative studies in Mathematics education is leading to improvement of educational process.

In present study, it was used a qualitative and quantitative study. According to Yin (2002) the qualitative and quantitative approaches are not excluded, but rather complement each other to have a better perception of the phenomenon under study. The study was performed considering the parameters described in the following lines.

3.1 USING PBL

According to Dewey (2016) Education must serve to solve life situations and educational action has as fundamental element the improvement of social relations, learning has to be practical in order to allow the development of skills in students. It turns out that this author already considered the need to learn through real situations and problems of life as in problem-based learning.

The use of PBL is recommended in engineering, particularly because it promotes deeper learning and develops problem solving skills, and bring significant improvements with regard to the development of general skills and promoting positive attitudes among the students (Yusof *et al*., 2005). That problem solving is not at all restricted to real world problems; problems emerging from the 'world of mathematics' can be very rich sources for problem solving activities.

What follows in this section is the result of an experiment. This experiment was conducted at UNIFEI in the second half of 2016.

3.2 THE TWO TEACHINGS

The author had a good opportunity in the second half of 2016. He taught Calculus 3 for two different classes. In one class, it was used the traditional lecture method. In the other class, the PBL methodology was the standard.

The traditional lecture classes is a method widely used not only worldwide, but also in Brazilian universities. Particularly in mathematics courses this method is practically the only one to be used. Following a worldwide trend, some math professors are adopting different methodologies in teaching, in particular the problem-based learning.

Barrows (1986) defines PBL as a method based on the principle of using problems as a starting point to stimulate the acquisition and integration of new knowledge. The PBL methodology develops the ability to identify the information needed for a specific application, where and how to obtain them, organize them into a meaningful conceptual framework and how to communicate it to others. Group work favors the development of learning communities, enhancing student learning. Students who learn concepts in the context in which they are used will better retain these concepts and apply them more appropriately.

Students, organized in small groups and assisted by a tutor, deal with problems that may be a description of phenomena or events based on real life. Such problems require an explanation in terms of their underlying mechanisms or processes or actions to be solved. The problem is, therefore, the starting point of the learning process.

The PBL is an instructional method that challenges the student to learn to learn, working cooperatively in groups in search of solutions to real problems (Duch, 1996). The PBL does not deny the importance of content learning. But it does not recognize the future utility of memorized content acquired in abstract contexts. On the other hand, it emphasizes the ability to acquire conceptual knowledge, as it is needed, using this knowledge during the learning process around the problem. Knowledge is built during problem solving. Depending on the degree of difficulty to solve the problem, further deepening the knowledge to solve it is necessary. This helps to consolidate knowledge (Magertson, 1997).

Let us turn attention to our experiment. In the class that was used the active methodology, the problems were used to stimulate students' curiosity and to begin learning concepts. In this way, students could be stimulated to think critically and analytically and also to find and use learning resources. Below we will see the students' reports. Most of the goals of Calculus 3 course were achieved and they were satisfied. In traditional teaching, concepts are transmitted initially. Then the applications are presented. In the PBL methodology the opposite happens. The starting point in the learning process is the problem. It has the function to motivate, focus, direct and initiate students' learning (Boud; Feletti, 1991). Students become builders of their knowledge through working with others, researching and searching for new knowledge.

The problems were chosen to be made on the same day of class. Many of UNIFEI students are in engineering. So the problem to be worked took that into consideration. Problems involving center of mass or center of gravity, volume calculations and surface areas, fluids, among others, were elaborated. Some problems had a direct application in engineering.

One of the main challenges is the design problem-solving tasks that are original, non-routine and new to the students. So that problem solving were not at all restricted to real world problems. Problems emerging from the ‘world of mathematics’ were a very rich sources for the problem-solving activities (Doorman et al., 2007). So the problem to be solved ranged from real-world problems and typical engineering problems to just mathematical problems.

In practice the following has been done. Some problems were passed to the groups. Everyone always worked with the same problem. After a while ideas began to emerge on how to solve the problem and what theoretical part applied. Some groups realized quickly what were the necessary mathematical foundations. This has motivated some students to always study before each class. When students were already familiar with the problem and most had ideas for solving it, the teacher briefly explained the mathematics involved in the problem and gave a simple example. At the end of each class, one group was chosen to present the solution to the whole class. If no group could solve the problem in one class, it was given the opportunity to solve it in the next class.

In the traditional teaching class, the concepts were presented initially. The theoretical part was detailed as much as possible. Then, the same problem used in the PBL class was presented and resolved. Then the teacher gave a few more examples. In the end, some students had their doubts or not.

3.3 THE TWO CLASSES

In the second half of 2016 the author has taught for two classes, A and B. Class A had 36 students. In the first week 6 groups were formed with 6 people each. As mentioned above, the goal was that in every class the students worked with a problem involving different topics content. The teacher took the doubts of the students and encouraged them to solve the problem as a group.

Class B had 48 students. After explaining the theoretical part, some examples and exercises were solved. At the end of each class, the students cleared their doubts, which in general were few.

Outside the classroom was a monitor. He is a student who had already taken Calculus 3 with a good grade. He had a weekly service schedule, about three times a week.

Here are three examples of problems designed for class A students. These problems were proposed to students during the semester, at different times. Some of the problems included applications in mathematics itself (how to calculate a volume) or in some engineering (mass center calculation). At the beginning of each class, the teacher briefly explained the mathematical content involved in the problem being proposed. Then an example of how to use the mathematical techniques involved was presented. The example was out of all practical application. Hence, the problem of the day was presented and explained to the groups. From that moment the teacher acted as a tutor, guiding the groups to the correct resolution. Most of the problems were solved the same day they were proposed. However, a few problems were proposed in one class, but the resolution was obtained in the next class.

A problem was passed to students with the goal of dealing with triple integrals, volume and change of variables (Stewart, J. 2006, p. 1034). The problem was: how much of ice cream do you have in the cone (volume)? The data provided were: a cone of ice cream was a cone and the ice cream was a part of a sphere. The teacher, acting as a tutor at that time, encouraged the groups to take the following steps. Initially, students should sketch the region in question. Then they should write the volume of the region. In this step, an important point was not to forget the Jacobian of the spherical coordinates. And finally, they should calculate the triple integral in question. This problem the students were able to solve in one class. One group was chosen to explain to all the step by step resolution.

As a second example of a problem worked by the class A groups, we have a typical engineering problem. A problem involved calculating the center of mass (or centroid) of a solid (Stewart, J. 2006, p. 1027). In this problem, the solid was given by a region of space bounded by a paraboloid and three planes. With this problem, students would learn how to sketch a region in space and use a triple integral. One question that arose was: why did not the problem provide the density of the material? With the help of the teacher, who acted as a tutor, not responding directly to the questions, the students understood the point in question. For a solid of constant density, the calculation of the center of mass does not need this data, that is, the density is indifferent to the result. This problem has been resolved on the same day it was proposed. The group that finished first was chosen to present the solution to everyone.

As the third and last example of problems proposed, we present a problem in which there were more difficulties of the students. This problem was proposed in one class, but the resolution was completed only in the next class. It was also a center of mass problem. However, it involved concepts of parametric surfaces (Stewart, J. 2006, p. 1108). The major difficulty was to apply the newly learned concept of parameterization of surfaces. At the beginning of the next class, the teacher explained some examples of parameterization of surfaces and identify the domain involved. After this, the students wrote the integral of the center of mass relative to each ordered axis. With the help of the teacher-tutor, the groups solved the integrals involved. Finally, a group was chosen to present the resolution for the class.

Each exam contained five questions. Each question sought to ascertain whether the student had learned the mathematical techniques of the content learned. The Exam 1 contained the following question. Given the density of a solid object, correctly write the integral that calculates the mass of that object. Another question of this Exam was: What is the volume of the interior of the ellipsoid?

The content covered in exam 1 was: double and triple integrals, regions in space, changes of variables and applications presented (center of mass, mass, volume, energy, moments of inertia, electric charge). The content covered in the exam 2 was: line and surface integrals, rotational and divergent, parametric surfaces, Green, Gauss and Stokes theorems and related applications explained (volume, circulation, flow, energy conservation, moment of inertia, centroid). Each question of the exam approached a content seen by the two classes. No question was of something new, which the students did not know. That way the exams were relatively easy.

In Exam 2 there was a question to calculate the line integral of a closed path. The goal was that the student used the Green's theorem. The exam questions in each class were basically the same. For example, this question to use Green's theorem used a closed path. In class A the closed path was an ellipse and in class B, the path was a circle with a different center from the origin. One question from exam 1 was to identify the integral region for a given integral. In class A this region was a cone and in class B, a sphere. Thus, there were no significant differences between the exams of each class. The exams were essentially the same.

3.4 THE TWO RESULTS

The results presented in the following first table are from common evaluations in the two classes. These evaluations include individual exams. Evaluative activities also include exercises and tests done through the university platform. The scores for the PBL activities corresponded to 20% of the final grade. For the traditional teaching class, the class B, these 20% were diluted in the exercises and tests. Thus, the individual exams had the same weight.

The exercises on the university platform (similar to Moodle) were proposed for both classes as a way to fix the content learned, to train the mathematical techniques involved in each specific content and to serve as a review for each exam. Such exercises were to be done through a computer (personal or university), individually and outside of class hours.

Table 1- Comparison of results (in %)

	A	B
Exercises 1	65,0	66,3
Exercises 2	63,9	59,8
Exercises 3	90,0	86,7
Exercises 4	86,7	83,3

Table 1- Comparison of results (in %)

Exam 1	53,3	52,5
Exam 2	52,1	24,3
Final grade	68,7	53,4
Frequency (missed classes)	10	11
Fail for absence	5,6	8,3
Fail grade	2,8	39,6

Exercises 3 and 4 were designed so that the students would train the mathematical techniques involved in the proposed problems. They also served as a basis for study and review for each test. Exercise 3 was performed two weeks before exam 1 and exercise 4 was done shortly before exam 2.

The score of exercise 4 included the commitment of the groups in problem solving (Ballela et al., 2014). There was also an average of personal evaluations and the teacher. These inclusions in the score of exercise 4 were made only for class A. Thus, for the class A, half of the score of exercise 4 included four group activities done throughout the semester.

For class B exercises 1 to 4 were just exercises done on the university platform. The exercises were to be solved through a computer and individually, outside the class time. These exercises addressed the mathematical techniques involved.

The evaluative methods used in class A were as follows. The Immediate Feedback Assessment Technique (IF-AT) was used twice. The team-based learning (TBL) was used in one class in order to have a more dynamic and group assessment. The problems solved by the groups were scored at the end of the semester. The activities in common with class B were the exercises done on the university platform and the individual exams. These exercises and the exams were the only evaluative activities of class B.

The Immediate Feedback Assessment Technique can promotes learning (Epstein et al., 2002; Bollela et al., 2014). The way to make use of this technique, one can access, for example, the site: https://www.uc.edu/content/dam/uc/cetl/docs/IF-ATinstructions.pdf

The Immediate Feedback Assessment Technique (IF-AT) was used twice. An analogy of this technique would be with a multiple-choice proof, and the answer was put into a kind of "scratch card". Exercises 1 and 2 were done using this IF-AT technique. Four exercises were proposed based on the problems solved previously. It has been noticed that some groups strive to guess the answer without actually solving the proposed question. So this technique was not used anymore.

The team-based learning (TBL) was used in one class in order to have a more dynamic and group assessment (Bolella et al., 2014). In practice the following has been done. Students know the content of each lesson and was expected to study preparing for each lesson. On the day of the class, the teacher explained the mathematical part involved, solving some example to fix the ideas. This part took about 30 minutes, but could be a little more depending on the content involved. Then a problem was passed to the formed groups. The teacher acted as a tutor, removing doubts from the groups and encouraging group discussion and the exchange of ideas among the students. At the end of each class a group was chosen to solve the proposed problem (usually the one that ended first). If no group solved the problem, the next lesson was the continuation so that the groups could solve. So that there would not always be a same group solving the problem at the end of the lesson, the group that solved the problem in one class would not solve the problem of the next class. This stimulated a co-operation of the components of the group and a concentration, since, generally, each group struggled to be the first to finish to solve the problem in the framework for every class.

Each TBL made by the groups contained three questions. In the first there was the following question: identify the inner region of the intersection of a plane and a paraboloid. In the second, there was the question: calculate the triple integral in Cartesian coordinates. The objective was to identify the surface in question (part of a sphere) and make a correct change of variables. All TBL questions were multiple choice, as explained above.

According to the rules of the university, for the student to be approved, a minimum score of 60% and a minimum frequency of 75% are required.

“Students who demonstrate a strong grasp of the material regardless of how many classes they attend should receive a passing grade”. Many teachers think and act like this. Some professors find that just because students aren’t in their seats doesn’t mean they can’t meet course objectives. We will not go into the merits of the controversial issue. But for the PBL class, the presence of the student is of great importance.

At the end of the course students filled out a form. So we can get feedback from them, as can be seen in Table 2. One student said: “The professor's attitude toward receiving student feedback is very good. It would be nice if it were done by all teachers to have a better education.”

In the completed form it was possible to have feedback from the students. Moreover, it was possible to have personalized feedback. The goal was to identify if they already knew any different methodology in teaching, satisfaction with the methodology adopted during the semester and study habits. In the final part of the form, there was an opportunity for the student to make a personal comment.

Table 2 - Students Feedback (in %)

	A	B
I like classes where students are forced to find part of the resolution of the exercises.	77,1	19,6
I do not like group activities.	0	-
I prefer lectures with professor explaining everything.	20	78,3
I struggled to concentrate in class and understand the content during the professor's explanation.	82,9	67,4
I studied before each class the day of the content.	14,3	2,2

Table 2 - Students Feedback (in %)

	A	B
I did not pay attention to the teacher and would rather entertain me with the phone.	0	6,5
I could not keep up with the sequence of course content and neither the professor's reasoning	17,1	43,5
I follow the content studied every week.	20	10,9
Usually, I study in the exam week.	60	60,9
I like to strive to do well in the course, but there are many activities at the university that prevent this.	54,3	58,7
I pose some questions to the professor.	45,7	21,7
I take my doubts to the monitor every week.	0	2,2
I take my doubts to the monitor during the exam week.	22,9	8,7
I do not have the habit of clearing my doubts with the monitor or the professor.	8,6	17,4

Let's look at some comments made by students in class A.

One student in class A, in the second week of the course, suggested a methodology that was adopted in an electrical engineering course. The engineer teacher recommended the students to do some exercises. In the following class this teacher based his class on these exercises. The students were evaluated regarding the resolution of some of the proposed exercises. Thus, the student studied more at home and was ‘forced’ to do several exercises before each class. This methodology seems to be good to apply in mathematics courses, but does not follow the PBL idea. In that, the objective is a group of students work together to obtain the solution of the problem. Each get the solution and then discuss, does not follow the PBL model.

To make clear to this student the methodology adopted in the Calculus classes that he was studying in the semester, the calculus teacher explained what the classes would be like. The goal was to solve a problem with the team it each class. However the student could (and even should) study in advance the mathematical concept addressed, even solving some exercises. Thus, because it is not the PBL methodology itself, the Calculus teacher did not adopt the teaching method of the engineering teacher, as suggested by the student.

Another student suggested: “I would like more exercises to be solved step by step by the teacher, because the Calculus 3 content is difficult, and we do not have to be afraid of the matter”. This same student said he likes the methodology adopted, but we are afraid of the new method. This particular student took one of the best grades in the class.

A student even suggested that each class have a list of exercises to be delivered. This comment was made by a student in class A, but more students in class B made this 'request'.

One student suggested going beyond the classrooms. He suggested applying the content to some practical project, such as the university contributing to the city.

Several students asked to review past content. Some students have recognized that the methodology is similar to that adopted in other countries, such as the United States. Exchange students had experiences of classes with the same methodology.

Regarding the evaluation method, some said to have the note of group exercises with different weight, so that the test had a lower weight in the final grade. Some even said that an individual exam is not the best method of evaluation.

Some have made positive comments to the teacher. “The teacher chose a methodology and followed it faithfully.” And: “Although I wanted more explanation on the board, the teacher helped the groups and encouraged the students to do the exercises and always clarified the doubts.”, and also “ the classes were fun”.

Now turn our attention to class B.

Several students in class B commented on what should improve: try to promote greater involvement of students in class; having group exercises or presenting seminars; having fewer students in the class would help learning.

Some realized that they could have been more engaged. For example, some wrote, “The content I consider to be of a difficult and complex level” or “I did not try as I should” and also “The content takes a lot of attention and often could not follow as I should”. A student admitted for those who work, it is more difficult to review the contents necessary for understanding the subject.

Regarding the different methodologies of teaching and learning, there were controversial comments. A student who says he knows the PBL methodology said that different methodologies should not be applied in calculus. More than one student commented on this. One student said: “In exchange, I studied PBL in Europe. It works there and I think it's different from the PBL applied here”. Also, another student said “I do not agree with active methodologies” and "The methodology adopted by the teacher was sufficient” even “I thought the way of teaching was very fair”, was commented.

Despite receiving some compliments, not everyone was satisfied with the teacher or the methodology. Some students made negative comments about the teacher or the way he taught. There was comment about the lack methodology and excitement to teach. Another said the professor should induce curiosity in students. One student even commented the following: “Calculus 3 should not be compulsory for all students” and another, “The content should be smaller”. Several students commented the teacher should resolve more exercises on the board.

Some said the University should try to approach calculus classes with engineering through projects. And another suggested there should be more dynamic classes, with students doing group exercises and then on the board as well. There should be classes activities with student participation.

4 DISCUSSION

Regarding evaluative methods, practically all were satisfied. In both classes, more than 90% of the students liked the evaluation methods. But there was an insistent request in class B: weekly evaluative exercises should be given to decrease the weight of the final exam.

Interestingly, 17% of class A said they knew some different methodology for teaching and learning, and 65% mentioned PBL. On the other hand, 39% of class B students said they knew about other methodologies and of these, 50% mentioned PBL.

With respect to class A, some data were interesting. About 37% of the students said they were more encouraged to study and sought to see practical applications of the content learned. In addition, 63% of students said they felt closer and more comfortable with the teacher, something they had not before. And about 25% said that they learned more from this methodology than they learned the other calculation contents (Calculus 1 and 2) and were able to have more interest in mathematics.

As can be seen in Table 2, only 20% of students in the class A said they prefer the traditional teaching. Talking to these students, the following was noted. Most of these were already working on internships. Therefore, these students had to miss several classes and, consequently, could not help the group to solve some of the problems. This situation bothered them and they preferred traditional lessons so they would not make any commitments to other classmates and not lose points for not participating. In addition, these students were keen to see the content lost through an expositive class, imagining that they would quickly learn the lost content. So the rate of 20% could be easily reduced. All students in this class said they enjoyed group activities.

A further notable factor is the following. The study habits of class A students were better. These habits include: following the sequence of content, asking questions with the monitor or the teacher, and focusing on the content of the subject on the day.

The vast majority of students in class A liked the methodology adopted. We realize that there is a certain fear of the students. It's like Newton's law of inertia. For several years these students only had contact with only one methodology, the traditional teaching through lectures. So they get bothered at first, wanting to always do something that includes lectures. But with an effort made gradually by them and by the teacher, the end result is very good, and really was. On the other hand, class B, following the principle of Newton's law, preferred the traditional lecture classes. They preferred what they knew.

Several students from the class ‘traditional’ said the content was too much and that the theorems (Green and Stokes) were difficult. They ended up creating a psychological problem in the matter. This did not occur with the class in which PBL was used. Many faced the problem to be solved as a challenge that they should overcome. In this way, they ended up looking at the content in a more practical way. Each content and theorem was for them a means to come to an end: solve the proposed problem. Students in PBL educational environment are more dedicated and productive. This, plus the presence of more efficient study habits, contributes to the development of socially and professionally desirable skills and attitudes.

Although many did not know the PBL, some suggested more dynamic classes. For example, one student of class B said Calculus courses should take lessons on the computer to view what is being learned. This suggestion is particularly interesting. This could be adapted for both traditional and PBL classes. New technological tools create new possibilities for problem solving in mathematics (Doorman et al., 2007). This has already been tested at the university for some math courses, but without success due to bureaucratic procedures. But it is possible that soon we will take some classes like this.

There are prospects that the PBL will assume great importance in the educational fields in Brazil, inasmuch as it seeks alternative methodologies aimed at the articulation of theory with professional practice, promoting real transformations in the current ways of teaching and learning (Mamede and Penaforte *et* al, 2001).

The increased demand for dedication is also a complicating factor for professors, who end up having less time to devote to more academically valuable activities (e.g., research). The procedural and dynamic nature of the PBL and the fact that it is difficult to close the planning in advance can lead teachers to see it as a hindrance. Another factor of frustration for some professors is the impossibility of covering all the content stipulated for the curriculum through problems. This is due to the fact that PBL is a methodology focused on the student and therefore fit them also the course of the study, which may differ from those previously planned by the tutors. In addition, the deepening in the subjects compromises the time that could be used to cover more contents. PBL also seems to cause some psychic discomfort in professors as testing their flexibility and their knowledge. Because the problems are open-end questions beyond the teacher's area of expertise can be raised by the students, especially in recent year.

We see from the tables presented that the performance of class A was better, especially in exam 2. The problem-solving methodology provided a better learning environment. We can say this by the way in which the doubts of the problems given to class A were solved. The teacher acted as a tutor, helping on the way to clear the doubts. He did not respond directly to doubts. In this way the students would discuss the concepts in groups and arrive at a solution. As we can see from the students' reports, the students in class A were more motivated to study the contents beforehand. Also, some students studied after each class. In this way, contact with the specific content of the subject was more regular than the students in class B. The effect of group discussions in each class was also positive. This is not to mention the stimulus provided by the problem itself. This has helped a more meaningful learning. As the teacher acted as a tutor, this provided closer contact with the groups and each student. As reported by the students, this was good because they felt more comfortable asking questions, not being afraid to "ask something very simple".

In contrast, class B did not have many stimuli to study regularly, even taking the exercises from the university platform. Few students took questions with the teacher. Most students studied only a few days before each exam. In class B the teacher presented more examples of each content. But he was the only one to speak and explain while the students listened attentively and passively. We can then conclude that an active attitude by the students can produce good results. Also a group discussion of practical problems and mathematical techniques is positive. In addition, applied problems can stimulate learning.

We did not conceive here a salvationistic attitude, as if the PBL methodology solved all the problems of Higher Mathematical Education. Thus, we do not have a reductionist view. On the contrary, the PBL methodology must be seen as an alternative to traditional teaching and can present better results. As presented, this methodology has some limitations. We can observe that some students reported that they preferred traditional teaching. Others said that there could be more explanations and examples of resolutions by the teacher. This also shows that it is also necessary for the students to leave the comfort zone and try harder to leave the passive attitude (seeing the teacher do everything) and to have an active attitude (going after the solution for himself and with the help of the group).

Some teachers still complain that the group work difficult the evaluation of individual students.

Although it is beyond this article to discuss the validity and the property of traditional evaluations, considered as objective, it is necessary to indicate that the systematic of evaluation of student performance in the PBL must be coherent with its principles. If this does not happen, there is a risk of conflicting messages regarding educational goals being passed on to students. It is known that students organize their time and efforts in order to achieve their personal goals (i.e., "passing" in the subject, getting the diploma). If told that the assessment is always individual, this could undermine the sense of teamwork.

5 CONCLUSION

Students who have had experience in doing some discipline with the PBL methodology have preferred this methodology. There are few students who prefer traditional classes, with the teacher explaining everything, having some exercise classes and then an individual exam to evaluate the knowledge acquired. The vast majority can see the benefits of applying a new methodology. That students realize that they can learn better and more depth in this type of active methodology. Some students felt more motivated to learn the content, preparing in advance for classes, trying to see practical applications of learning content. Many students ended up clarifying their doubts with the teacher more than they did before and felt more at ease in doing so.On the other hand, students who participate in courses with traditional teaching methods, with lectures and teacher explaining everything, end up having a more passive attitude. Few of these students asked the teacher questions during class. Many students prefer the traditional method of lectures with the teacher explaining everything. But they are students who have not had an experience of classes with different methodologies. Experience has shown that when a student has the opportunity, he or she prefers the PBL methodology.

Perhaps the PBL methodology will not solve all the problems of Calculus teaching in the universities of Brazil and the World. But it's certainly a big step in helping to have students more engaged with learning and more satisfied with what they do.

The way students view the professor changes. The PBL methodology allows a greater approximation and iteration between teacher and student. This was noted in the students' comments. The students of class A did not give any negative comments regarding the methodology or the teacher. On the contrary, many said they enjoyed and praised the teacher. On the other hand, some students in the traditional teaching class B made some negative comments about methodology and the teacher.

Individualistic, competitive and introverted students may not adapt to the participatory and collaborative nature of learning with this methodology. However, attention must be paid to the fact that the skills and attitudes promoted by the PBL are necessary for all professionals, regardless of their personalities.

A challenge to improve teaching with the PBL methodology is to achieve even more motivating and applied problems for students. To develop good problems for students, some guiding questions in the design of activities are important (Code et al., 2014). These guidelines may be, for example, "Are students reflecting on their work, possibly by having to explain choices to others?" or "Are students practicing the skills outlined in the objectives?".

Traditionally, universities are structured by departments supported around specific disciplines. The PBL method is characterized as interdisciplinary. Teaching through the PBL may seem inefficient due to the number of students, but there can be no confusion between efficiency and effectiveness that engenders learning. Nevertheless, the application of this educational method must be a departmental or institutional decision (Wood, 2004). Also, it is not so simple to encourage more professors to be able to innovate their teaching methods, leaving the comfort zone, many still prefer to continue with the traditional methodology.

The advantage of the PBL most cited in the literature is certainly its ability to make learning more dynamic and enjoyable, shared by both students and teachers. This in itself can greatly contribute to instill in students an appreciation for the study and, consequently, the willingness to learn lifelong learning. The PBL also fosters a learning environment where there is more camaraderie. It stimulates the establishment of partnerships between students and between students and teachers and the development of communicative and social skills. Another advantage is the ease with which curricula can be (conceptual, procedural and attitudinal) assessed as totally or partially irrelevant to professional practice by students, faculty and coordination.

Research on the PBL shows an undoubted positive appreciation of students and teachers. The results of the study and presented above also show that students develop better study habits. PBL students usually study more for understanding than for memorization, and demonstrate more focus and organization in their efforts. As they study content every week, even if only in classrooms, they become more familiar. So, do not be afraid to study a certain topic.

The data analyses clearly demonstrates that PBL can be used in mathematics curricula (at least for calculus courses), even in non-curricular deployments. It seems that even in partial deployments, the gains obtained justify its adoption. However, it must be reiterated that this methodology does not solve all the dilemmas of higher education. It is only one of the alternatives to teaching based on the transmission and reception of knowledge model.

Despite the efforts of SBEM, STEHM, and COBENGE, active methodologies in teaching mathematics in higher education have been few, at least until the present time. But a good effort has been made by Brazilian universities and teachers motivated to improve teaching and learning. The result can be encouraging, as shown by the result of the research done.

The present study contributes to society, especially in new universities, in developing countries that are still trying to match their educational processes, in the sense that it is possible to improve mathematical education, making it more interesting for students, by humbly seeking Learning through interaction with older and more experienced institutions, and the use of methodologies such as PBL, which are not so new but well applied, can lead to reduced dropout and improved mathematical learning.

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